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F.E. (Revised 2003 Course)

ENGINEERING MATHEMATICS - II

Time : 3 Hours]

[Max. Marks :100

Instructions to candidates:

- i) Answers to the two sections should be written in separate answer books.
- ii) In section I, attempt Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6. In section II, attempt Q.7 or Q.8, Q.9 or Q.10, Q.11 or Q.12.
- iii) Neat diagrams must be drawn whenever necessary.
- iv) Figures to the right indicate full marks.
- v) Use of non-programmable electronic pocket calculator is allowed.
- vi) Assume suitable data, if necessary.

SECTION - I

Q1) a) Solve (ANY THREE): [12]

i)  $x^4 \frac{dy}{dx} + x^3y - \sec(xy) = 0.$

ii)  $(y^4 - 2x^3y)dx + (x^4 - 2xy^3)dy = 0.$

iii)  $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0.$

iv)  $(y - 2x^3)dx - x(1 - xy)dy = 0.$

b) Form the differential equation whose general solution is  $y = ae^{-2x} + be^{-3x}$  [5]

OR

Q2) a) Obtain the differential equation whose general solution is  $y = A \cos(\log x) + B \sin(\log x).$  [5]

b) Solve (ANY THREE): [12]

i)  $y(2x^2y + e^x)dx = (e^x + y^3)dy.$

ii)  $\frac{dy}{dx} = \frac{x + 2y - 3}{3x + 6y - 1}$

iii)  $\cos y - x \sin y \frac{dy}{dx} = \sec^2 x.$

iv)  $\frac{dy}{dx} = (x - y + 1)^2 + (x - y).$

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Q3) Solve any THREE of the following:

A particle of mass  $m$  moves in a horizontal straight line OA with an acceleration  $\frac{mk}{r^3}$  at a distance  $r$  and directed towards O. If initially the particle was at rest at a distance 'a' from O, find the distance of particle from O at the end of time  $\frac{a^2}{2} \sqrt{\frac{3}{k}}$ . [6]

ii) The equation of an L - R circuit is given by

$$L \frac{di}{dt} + Ri = \sin 10 t.$$

If  $i = 0$  at  $t = 0$  find the expression for  $i$  in terms of  $t$ . [5]

iii) A particle executes SHM. When it is 2 c.m. from mid path its velocity is 10 cm/sec. and when it is 6 cm. from the centre of its path, its velocity is 2 cm/sec. Find the period and its greatest acceleration. [6]

iv) If 30% of a radio active substance disappeared in 10 days, how long will it take for 90% of it to disappear. [5]

OR

Q4) Solve the following (Any THREE):

i) A pipe 30 cm. in diameter contains steam at  $150^\circ\text{C}$  and is protected with a covering 5 cm. thick for which  $k = 0.0025$ . If the temperature of the outer surface of the covering is  $40^\circ\text{C}$ , find the temperature at a point which is at 16.5 cm. from the axis of the pipe. [6]

ii) A constant electromotive force  $E$  volts is applied to a circuit containing inductance  $L$  henries. If the initial current is zero, find the time that lapses before the current reaches 25% of its maximum value. [5]

iii) A body of mass  $m$  falls from rest under the influence of gravity and a retarding force due to air resistance proportional to the square of the velocity. Find the velocity and distance described as function of time. [5]

iv) Water at temperature  $100^\circ\text{C}$  cools in 10 minutes to  $80^\circ\text{C}$  in a room having temperature  $25^\circ\text{C}$ .

Find

- temperature of water after 20 minutes,
- the time required to lower the temperature of water to  $40^\circ\text{C}$ . [6]

Q5) a) A sphere of radius 3 passes through the origin and meets the coordinate axes in A, B & C. Find the locus of centroid of the triangle ABC. [5]

- b) Find the equation of the right circular cone which passes through the point (2, -2, 1) with vertex at the origin and axis parallel to the line

$$\frac{x-2}{5} = \frac{y-1}{1} = \frac{z+2}{1} \quad [6]$$

- c) Find the equation of right circular cylinder of radius 2 and whose axis lies along the straight line

$$\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5} \quad [5]$$

OR

- Q6) a) Find the equation of the cylinder whose generators are parallel to the line

$$\frac{x-1}{3} = \frac{y-1}{1} = \frac{z+2}{\sqrt{6}}$$

and whose guiding curve is,

$$x^2 + y^2 = 25, \quad z = 0. \quad [5]$$

- b) Find the equation of the cone whose vertex is at the point (1, 1, 3) and which passes through the ellipse  $4x^2 + z^2 = 1, y = 4.$  [5]  
 c) Find the equation of the sphere which passes through the point (3, 1, 2) and meets XOY plane in a circle of radius 3 with centre at (1, -2, 0). [6]

**SECTION - II**

- Q7) a) Find a Fourier series for.

$$f(x) = x + \frac{x^2}{4} \text{ in } (-\pi, \pi) \text{ and } f(x+2\pi) = f(x)$$

Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots = \frac{\pi^2}{12}$$

- b) If  $u_n = \int_0^{\pi/4} \sin^{2n} x \, dx$

prove that  $u_n = \left(1 - \frac{1}{2n}\right) u_{n-1} - \frac{1}{n \cdot 2^{n+1}}$

- c) Prove that  $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \int_0^{\pi/2} \sqrt{\sin \theta} \, d\theta = \pi$

OR

Q8) a) The following table gives variation of periodic current over a period.

t (secs) :	0	T/6	T/3	T/2	2T/3	5T/6	T
A (amp) :	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amperes in variable current and obtain amplitude of 1<sup>st</sup> harmonic. [8]

b) Evaluate  $\int_0^{\infty} x^n e^{-x^m} dx$  [4]

c) If  $I_n = \int_0^{\pi/4} \sec^n \theta d\theta$  OR

prove that  $I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{(n-2)}{n-1} I_{n-2}$

hence evaluate  $\int_0^{\pi/4} \sec^6 \theta d\theta$  [5]

Q9) a) Prove that [5]

$$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} dx = \log \frac{b}{a} \quad \begin{matrix} a > 0 \\ b > 0 \end{matrix}$$

b) Trace the curves (Any 2): [8]

i)  $x^2 y^2 = a^2 (y^2 - x^2)$

ii)  $r = a \cos 3\theta$

iii)  $x = t^2, y = t - \frac{t^3}{3}$

c) Show that  $\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2} [\text{erf}(b) - \text{erf}(a)]$ . [4]

OR

Q10) a) Prove that [4]

$$\text{erf}_c(-x) + \text{erf}(x) = 2.$$

b) Trace the curves (Any 2):

i)  $r = \sqrt{2} + \cos \theta$

ii)  $y^2(2a - x) = x^3$

iii)  $x^4 + y^4 = 2a^2 xy$ . [8]

- c) Find the arc length of the cycloid  $x = a(\theta + \sin\theta)$ ,  $y = a(1 - \cos\theta)$  from one cusp to another cusp. [5]

Q11)a) Evaluate  $\int_0^{\infty} \int_0^{\infty} e^{-x^2(1+y^2)} x dx dy$  [5]

- b) Find mean value of  $x^2y^2z^2$  over positive octant of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad [5]$$

- c) Find the area between the curve  $y^2x = 4a^2(2a - x)$  and its asymptote. [6]

OR

Q12)a) Evaluate  $\iiint x^2yz dx dy dz$  throughout the volume bounded by the plane

$$x = 0, y = 0, z = 0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1. \quad [5]$$

- b) Find the volume enclosed between the cylinders

$$x^2 + y^2 = 2ax \quad \text{and} \quad z^2 = 2ax. \quad [5]$$

- c) Find moment of inertia of one loop of lemniscate  $r^2 = a^2 \cos 2\theta$  about initial line. [6]

